

GENERAL MATHS HSC

MODELLING
NONLINEAR
FUNCTIONS

NAME: _____

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HSC CAPACITY MATRIX - GENERAL MATHEMATICS

CONTENT	CAPACITY	GRADE 10	GRADE 11	GRADE 12
1. Quadratic functions of the form $y = ax^2 + bx + c$, where $a \neq 0$, and their graphs. The vertex, axis of symmetry, and direction of opening.	10-11	✓	✓	✓
2. Cubic functions of the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, and their graphs. The y-intercept, x-intercepts, and direction of opening.	10-11	✓	✓	✓
3. Exponential functions of the form $y = a \cdot b^x$, where $a \neq 0$ and $b > 0$, $b \neq 1$.	10-11	✓	✓	✓
4. Hyperbolic functions of the form $y = \frac{a}{x}$, where $a \neq 0$.	10-11	✓	✓	✓
5. Graphical solution of systems of linear and quadratic equations.	10-11	✓	✓	✓
6. Graphical solution of systems of linear and cubic equations.	10-11	✓	✓	✓
7. Graphical solution of systems of linear and exponential equations.	10-11	✓	✓	✓
8. Graphical solution of systems of linear and hyperbolic equations.	10-11	✓	✓	✓

Year and
What was the most important thing you learned?
What was something new you learnt?
What part(s) of this task did you find to be difficult?

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THE PARABOLA

$y = ax^2 + bx + c$

If a is negative
concave down

If a is positive
concave up

If a is a small value
skinny

If a is a large value
skinny

If h is negative
Parabola slides to the right

If h is positive
Parabola slides to the left

$y = (x-h)^2$

eg
Consider the quadratic function $y = x^2 - 4x + 7$.
a) Complete the table of values below.
b) Graph the function for $x \geq 0$.
c) State the minimum value of $y = x^2 - 4x + 7$.

STOP

LOOK

THINK

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6

5

4

3

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-1

-2

-3

-4

-5

0

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Graph the function $y = 1 + 4t - t^2$.

STOP

LOOK

THINK

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eg A ball is thrown in the air. Its height, after t seconds can be given by the formula $h = 20t - 5t^2$.
a) Graph the function below.
b) From your graph determine the maximum height the ball will reach.
c) From your graph determine the total distance the ball has travelled in 4 seconds.
d) From your graph determine the time when the ball has reached 14 metres in height.

30

28

26

24

22

20

18

16

14

12

10

8

6

4

2

0

0

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NONLINEAR GRAPHS

Cubic functions: $y = x^3$

Cubic functions: $y = -x^3$

Cubic functions: $y = x^3$ for $x \geq 0$

Hyperbolic functions: $y = \frac{1}{x}$

Hyperbolic functions: $y = -\frac{1}{x}$

Hyperbolic functions: $y = \frac{1}{x}$ for $x > 0$

Exponential functions: $y = 2^x$

Exponential functions: $y = (0.5)^x$

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eg Graph the function $y = 2x^3$ by first completing the table of values.

x	-1	0	1	2	3
y	-2	0	2	16	54

60

55

50

45

40

35

30

25

20

15

10

5

0

-5

-5

-4

-3

-2

-1

0

1

2

3

4

5

*Multiplying $2x^3$ by a whole number makes it steeper.
*Multiplying $2x^3$ by a fraction makes it shallower.

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eg Graph the function $y = -2^x$ by first completing a table of values.

x	-2	-1	0	1	2	3	4
y	-4	-2	-1	-2	-4	-8	-16

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eg Gav invests \$10 000 at 8% p.a. with interest compounded annually. The growth pattern can be given by the exponential function $A = 10 000(1.08)^n$, where n is the number of years of the investment and A is the amount to which the investment grows.

a) Complete the table of values.

b) Plot your values on the plane provided.

c) From your graph, determine the number of years it would take for Gav's investment would grow to double his original investment.

n	0	1	2	3	4	5	6	7	8	9	10
A	10000	10800	11664	12587	13605	14726	15961	17324	18830	20481	22287

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Variation

INVESTIGATION: Consider the formula $y = 3x$. What value never changes? This means that y is in proportion to the value of x, that is, y is always triple x. So we could write $y = 3x$ as $y \propto x$

where \propto means "is in proportion to" and simply means that there is a constant value multiplying x (in this case, 3). The constant of variation (in this case, 3) is also the gradient of the straight line graph that represents the relationship between the two quantities.

$y \propto x$

That sign means "is in proportion to"

You simply replace it with " $= kx$ "

Consider the formula for the area of a circle. What value never changes? This means that the Area is in proportion with the square of the radius. So we could write $A = \pi r^2$ as _____

To solve direct variation questions:

1. Replace the \propto sign with " $= kx$ ";
2. Substitute all known values and solve for k;
3. Rewrite the original equation with the value of k subbed.

Now answer the question!

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eg If y varies directly as x and $y = 8$ when $x = 12$, find k and write an equation that expresses this variation.

eg If y varies directly as x and $y = 24$ when $x = 16$, find y when $x = 12$.

eg If t varies directly as the cube of w, and $t = 1728$ when $w = 6$, find the value of t when $w = 4.5$.

eg The surface area of a cube varies directly with the square of the length of the cube's edge.

a) A cube of edge length 5.5cm has a surface area of 181.5cm². Find their constant of variation.

b) Hence calculate the surface area of a cube with an edge length of 7.2cm.

Practise using this site:

http://www.algebraohh.org/lessons/lesson.aspx?file=Algebra_LinesTgDirectVariation.pdf

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STRANGE QUESTIONS!??!

Use the U-turn approach!

eg A new car has a fuel consumption of 7.2L/100km.

a) How much fuel is required for a journey of 3 600km?

b) The car has a petrol tank which holds 75L. How many km can it do on a single tank (on average)?

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eg Research conducted by a physiotherapist has determined that the height-to-mass rate (in cm/kg) of adult males is 2.26. Use this information to predict:

a) The height of a 75 kg adult male

b) The mass of a 195cm adult male.

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inverse variation

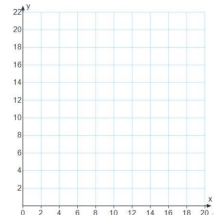
If 2 quantities VARY INVERSELY then an increase in one quantity results in a decrease in the other quantity and vice versa.
Inverse variation is represented graphically as hyperbolas.

$y \propto \frac{1}{x}$
 $y = \frac{k}{x}$

WHY?
Because $k \times \frac{1}{x}$

eg y varies inversely with x, and y = 10 when x = 2.
a) Find the constant of variation, k and hence the rule relating x and y

b) Plot a graph of the relationship between x and y for positive values of x.



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eg When a force is applied to a certain object, its acceleration varies inversely as its mass. When the acceleration of an object 12 m/s^2 , the corresponding mass is 3 kg .

a) Find the constant of proportionality.

b) Find the rule relating acceleration and mass.

c) Find the acceleration of a 1.5 kg object.

d) Find the acceleration of a 6 kg object.

eg The intensity of light (I) on a screen varies inversely as the square of the distance (D) between the screen and the light source. If the screen has 24 units of illumination when the source is 4 m away, determine the illumination when the screen is 6 m away.

eg The time taken to complete a job varies inversely as the number of workers. If 16 workers could complete a task in 6 days, the time taken for 25 workers working at the same rate is closest to:

A 2.67 days

B 3.84 days

C 7 days

D 9.375 days

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eg Peter notices that his 15 finches took 6 days to empty their seed container. He added 3 more finches to the cage. How long would it take the finches to consume the same quantity of seed as they did before the 3 were added?

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GRAPHS OF PHYSICAL PHENOMENA

<http://www.xpilot.co.uk/kg/> Soccer and Graphs

SMARTBOARD PAGE: physical_phenomena – SMART Notebook

In many cases, an algebraic function can be used to graph a physical situation.

eg The population of an island is increasing exponentially. The population of the island is modelled using the formula $P = 40000(1.03)^t$, where P is the population and t is the time in years.

a) What is the initial population of the island? (This is the value of P when $t=0$).

b) Complete the table of values:

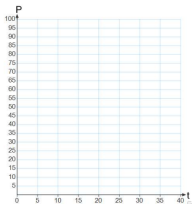
t	0	5	10	15	20	25
P						

c) Neatly sketch the population graph.

d) Use your graph to estimate the population after 18 years.

e) Estimate the time taken for the population to reach $59\,000$.

f) Extrapolate your graph to obtain an estimate of the time taken for the population to reach $100\,000$.



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eg A cinema owner believes that more people will attend the movies on cold days and so believes the number of people attending each session of the movies varies inversely with the temperature of the day.

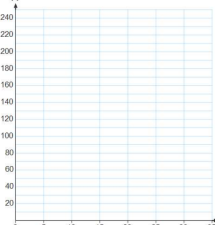
When the temp is 15°C , 80 attend a movie. Given that the cinema has a maximum of 120 seats and the cinema owner believes that a minimum of 40 people will attend, regardless of temperature.

a) Write an equation connecting the number of people attending the movie, N , with the temperature, T .

b) Complete the table of values below:

T	5	10	15	20	25	30	35
N							

c) Graph the relationship between attendance and temperature.




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
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
PAST HSC QUESTIONS


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6 Which of the following graphs best represents the equation $y = a^x$, where a is a positive number greater than 1 ?

(A) 

(B) 

(C) 

(D) 

Q28

(a) The air pressure, P , in a bubble varies inversely with the volume, V , of the bubble.

(i) Write an equation relating P , V and a , where a is a constant. 1

(ii) It is known that $P=3$ when $V=2$. 2

By finding the value of the constant, a , find the value of P when $V=4$.

(iii) Sketch a graph to show how P varies for different values of V . 2

Use the horizontal axis to represent volume and the vertical axis to represent air pressure.

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2010 HSC

13. The number of hours that it takes for a block of ice to melt varies inversely with the temperature. At 10°C it takes 8 hours for a block of ice to melt.
How long will it take the same size block of ice to melt at 12°C?

(A) 3.2 hours
(B) 20 hours
(C) 26 hours
(D) 45 hours

Q28

(iv) The company wants to produce a can with a volume of 1570 cm³, using the least amount of metal. Monica is given the job of determining the dimensions of the can to be produced. She considers the following graphs.

Surface area of a can with a volume of 1570 cm³

Dimensions of a can with a volume of 1570 cm³

What radius and height should Monica recommend that the company use to minimise the amount of metal required to produce these cans? Justify your choice of dimensions with reference to the graphs and/or suitable calculations.

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$y = \frac{1}{x}$

$y = 1 \div x$

subst 2: $y = \frac{1}{2}$

8: $y = \frac{1}{8}$

$1 \div \frac{1}{2} = 2$

$1 \times \frac{1}{2} = \frac{1}{2}$

subst $\frac{1}{2}$: $y = \frac{1}{\frac{1}{2}} = 2$

subst $\frac{1}{8}$: $y = 100$

$\frac{2}{3}$: $y = \frac{3}{2}$

Jun 5-10:25 AM